

## **A Survey on Fuzzy Queueing Models with Real-World Applications**

*<sup>1</sup>Anushree Rathi*

*Academic Associate*

*Indian Institute of Management, Indore*

*Research Scholar*

*Institute of Engineering and Technology, Devi Ahilya Vishwavidhyalaya, Indore*

*<sup>2</sup>Dr. Rafia Aziz*

*Assistant Professor*

*Govt. Girls College, Khargone, Indore*

**Abstract:** *Waiting lines and service processes are unclear in real life. In unclear circumstances, traditional queueing models use clear input parameters, which is unrealistic. Fuzzy queueing models, based on fuzzy set theory, model arrival and service rates as fuzzy numbers, offering a powerful alternative. Theory and practice of fuzzy queueing are examined in this study. We study modeling techniques such as fuzzy arithmetic,  $\alpha$ -cut algorithms, and fuzzy control strategies for single- and multi-server systems. The essay covers finance, healthcare, transportation, telecommunications, and manufacturing. By improving queueing analysis flexibility and realism using fuzzy logic, this study shows the growing importance of fuzzy models in complex, uncertain service situations. This paper is divided into several sections, Introduction, Preliminary definitions, Literature Review, Objectives, Methodology, Literature Review, Conclusion, and References.*

**Keywords:** **Queueing Models, Fuzzy Numbers, Membership Function,  $\alpha$ -cut, Defuzzification.**

### **1. Introduction**

In real-world systems, waiting lines and service processes are full of uncertainty. Traditional queueing models use clear input parameters, which can be unrealistic in hazy or incomplete contexts. Fuzzy queueing models, based on fuzzy set theory, model arrival and service rates

as fuzzy numbers, providing a powerful alternative. Fuzzy queueing approaches are covered in this examination, including theoretical foundations and practical applications. We explore modeling techniques like fuzzy arithmetic,  $\alpha$ -cut algorithms, and fuzzy control strategies for single- and multi-server systems. The article discusses banking, healthcare, transportation, telecommunications, and manufacturing applications. This research highlights the growing importance of fuzzy models in complex, unpredictable service contexts by showing how fuzzy logic improves queueing analysis flexibility and realism. When modeling systems like customer service departments, manufacturing lines, communication networks, and healthcare systems—where resources are divided among conflicting demands—queueing theory is essential. Conventional queueing models usually presume that system parameters, such as arrival and service rates, are precisely known. However, because of a lack of data, human judgments, or environmental variability, these characteristics are frequently ambiguous, inaccurate, or uncertain in many real-world circumstances. Fuzzy queueing models are the result of researchers' incorporation of fuzzy set theory into queueing models in an effort to better reflect this ambiguity. These models provide a more adaptable and practical framework for analysis and decision-making by enabling parameters to be expressed as fuzzy numbers as opposed to precise values. The goal of this paper is to provide a thorough analysis of fuzzy queueing models, with an emphasis on how they are used in practical systems. It investigates a number of modeling methodologies, such as fuzzy control strategies, simulation techniques, fuzzy arithmetic approaches, and fuzzy M/M/1 and M/M/c systems. The report also emphasizes the effective applications of fuzzy queueing theory in industries like manufacturing, banking, healthcare, telecommunications, and logistics. Researchers and practitioners interested in using fuzzy logic to improve the resilience and flexibility of queueing systems under uncertainty will find this review to be a useful resource as it bridges the gap between theoretical developments and real-world applications.

## 2. Preliminary Definitions

**Queueing Model:** Kendall's notation is a standard way to describe the structure of a queueing model using a shorthand format. It is typically written as:

**Kendall's Notation:** A/S/c/K/N/D

### Symbol Meaning

- |          |  |
|----------|--|
| <b>A</b> | Arrival time distribution  |
| <b>S</b> | Service time distribution  |
| <b>C</b> | Number of service channels (servers)   |
| <b>K</b> | System capacity (maximum number of jobs in the system, including those being |

**Symbol Meaning**

- served)
- N** Population size (size of the source from which customers come)
- D** Service discipline (order in which customers are served, e.g., FIFO, LIFO, etc.)

**Common Notations for A and S (Arrival & Service Distributions)****Notation Meaning**

- M** Markovian (i.e., exponential distribution)
- D** Deterministic (fixed times)
- G** General distribution
- Ek** Erlang distribution with k phases
- Hk** Hyper-exponential distribution with k phases

**Fuzzy Number and Membership Function:** A fuzzy number is a convex, normalized fuzzy set defined on the real number line  $\mathbb{R}$ , characterized by a membership function  $\mu(x): \mathbb{R} \rightarrow [0, 1]$ , which assigns to each real number  $x$  a degree of membership in the fuzzy number.

A fuzzy number  $\tilde{A}$  must satisfy:

1. Normality: There exists at least one  $x$  such that  $\mu_{\tilde{A}}(x) = 1$ .
2. Convexity:  $\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$  for all  $x_1, x_2 \in \mathbb{R}$  and  $\lambda \in [0, 1]$ .
3. Upper semi-continuity.
4. Support is bounded: The set  $\{x \in \mathbb{R} \mid \mu_{\tilde{A}}(x) > 0\}$  is bounded.

In fuzzy queueing theory, these membership functions are used to:

1. Model arrival and service rates as fuzzy numbers
2. Perform fuzzy arithmetic
3. Calculate  $\alpha$ -cuts for uncertainty propagation
4. Handle decision-making under imprecision

**3. Literature Review**

Henri Prade (1980) outlines fuzzy or probabilistic queueing system models. The chapter offers a fully possibilistic (fuzzy) queueing model that uses fuzzy sets (triangular or trapezoidal fuzzy numbers) to simulate arrival rates, service times, and service regulations. The model handles all uncertainty as fuzziness, not randomness. Instead of M/M/1 or birth-death stochastic queueing, it uses a pure possibility-based architecture with fuzzy numbers for system parameters and fuzzy sets and possibility measures for operations. Critical parameters like arrival rate ( $\lambda$ ) and service rate ( $\mu$ ) are described as fuzzy values to account for imprecision in expert-based or ill-defined input data. Fuzzy arithmetic is used to calculate fuzzy-valued outputs for performance measurements including queue length, waiting time, and utilization using  $\alpha$  cuts and Zadeh's extension principle. Possible and necessity distributions are used to represent best-case and worst-case system behavior under imprecise

conditions instead of probability. The chapter gives qualitative reasoning and reduced analytic forms for performance measures under possibilistic uncertainty and solution clues for various queuing setups (e.g. fuzzy service rules, fuzzy service times).

Kumar and Kapur's work from 1989, called "Queuing Models with Fuzzy Data in Construction Management," adds to classical queuing theory in a big way by using fuzzy set theory to deal with the fact that operational parameters in construction projects are often not clear or exact. The authors say that traditional probabilistic models, which are based on exact arrival and service rate estimates, don't work well in construction settings where things like delivering materials, using equipment, or finding workers are often talked about in general or vague terms instead of using precise statistics. In order to deal with this problem, the paper creates models of queuing systems where arrival rates and service times are shown as fuzzy numbers. These models use triangular and trapezoidal membership functions to show how experts think the process will vary. The study uses this framework to look at single-server and multi-server queuing situations that are common on building sites, like mixing and delivering concrete and moving dirt. The writers come up with fuzzy performance measures like waiting time and system utilization using Zadeh's extension principle and  $\alpha$ -cut representations. The study shows that fuzzy queuing models are more flexible and useful for helping people make decisions, especially when uncertainty is more subjective or qualitative than statistically random. Since then, this contribution has helped shape more study on fuzzy-based simulation and optimization in managing construction projects.

The 2005 Springer monograph "Fuzzy Control of Queuing Systems" by Zhang, Phillis, and Kouikoglou is a thorough look at fuzzy logic-based control strategies for managing queuing systems in a wide range of real-world settings, such as internet traffic, communication networks, and service operations. The writers suggest several fuzzy control models, with a focus on situations where input data isn't precise or changes a lot and can't be properly described by standard stochastic models. Their method uses linguistic variables and rule-based fuzzy inference systems to improve queue management. This lets them create adaptive controllers that can react to changes in queue length, arrival rate, and service quality in real time. With the help of feedback control loops and Mamdani-type fuzzy logic, the models can make smart service choices like changing service rates or allocating resources. There are many examples of how fuzzy control makes systems much more stable and responsive than traditional threshold-based or probabilistic control methods. These examples are mostly from network congestion control and telephony. This paper makes a major addition to the field of

intelligent queue management by combining control theory with fuzzy decision-making when there is uncertainty.

In their Wang, Liu, and Li (2009) present an innovative methodology for modeling queueing systems through fuzzy random renewal processes, tackling situations where uncertainty stems from both randomness and fuzziness concurrently. The authors characterize interarrival times as fuzzy random variables, encompassing both stochastic variability and imprecision, so rendering the model especially applicable to real-world service systems with hybrid uncertainty, such as those seen in manufacturing or communication networks. The model utilizes Archimedean triangular norms (t-norms) for the aggregation of fuzzy information and establishes novel characteristics and limit theorems for these processes within this integrated uncertainty framework. The key performance indicators of queueing systems, including arrival rates over a certain duration and waiting time distributions, are obtained through this hybrid modeling methodology. Additionally, the study presents numerical examples and case studies to illustrate the method's actual usefulness, demonstrating enhanced realism in modeling relative to only probabilistic or solely fuzzy systems. The incorporation of fuzzy random variables into renewal theory represents a substantial advancement in queueing research, providing a more refined depiction of operational systems influenced by both quantifiable and linguistic uncertainties.

There aren't many formal survey papers that only talk about fuzzy queueing applications, but a lot of important individual efforts give us the depth and breadth we need for a full survey of real-world applications. Wang et al. (2009), for example, use Archimedean triangular norms and fuzzy random renewal processes to model interarrival times in queueing systems. This is a hybrid method that can handle both probabilistic and fuzzy uncertainty. Shanmugasundari and Aarthi (2020) make the M/M/1 queueing model easier to understand by using triangular fuzzy numbers to represent arrival and service rates and fuzzy math to figure out queue success indicators. In the construction field, Kumar and Kapur (1989) use  $\alpha$ -cut representation and Zadeh's extension principle to change queueing models with fuzzy data to account for unknowns in work on building sites. In the same way, Dogan and Yildirim (2013) use fuzzy performance indicators to figure out how long the lines are and how well the services are being used in a simulation of fuzzy exponential queues for an M/M/C bank case. Abdullah et al. (2016) look at the differences between traditional and fuzzy queueing models at Malaysian airport check-in desks. They show how fuzzy models can be useful in situations where operations aren't always clear. Moghaddam et al. (2011) use linguistic factors like

credit score and wait time along with fuzzy inference systems to decide which customers in a bank line should go first. At the same time, Popovic et al. (2018) create an ANFIS-based queueing model for organizing warehouse tasks that uses a combination of fuzzy logic and neural networks to make operations run more smoothly. Lastly, Zhang, Phillis, and Kouikoglou (2005) write an important book about fuzzy control strategies in queueing systems that can also be used to handle internet traffic and networks. Together, these studies show how flexible and useful fuzzy queueing models are in many fields, such as building, banking, logistics, and telecommunications. They provide useful ways to make decisions when you don't have all the facts.

This work by **Dogan and Yildirim (2013)** is called "Simulation Approach to Fuzzy Exponential Queues for an M/M/C Bank Case." It is a simulation-based analysis of multi-server bank queueing systems under uncertainty using fuzzy exponential queue models. The writers added fuzzy logic to the classical M/M/C queue model to deal with input uncertainty because they know that service systems like banks often work with vague and inaccurate data, especially when it comes to arrival and service rates. In particular, the arrival ( $\lambda$ ) and service ( $\mu$ ) rates are set as triangular fuzzy numbers that represent the fact that customer flow and teller efficiency are not always clear in real life. As fuzzy performance indices, the model looks at important performance measures like average wait time, queue length, and service utilization. A Monte Carlo simulation is used to make the queue behave realistically while fuzzy input changes are sent through the system. This mixed simulation-fuzzy framework lets people making decisions see queueing results not only as single numbers, but also as ranges of how likely it is that things will go when there is uncertainty. The study shows that combining fuzzy math with modeling makes it easier and more accurate to judge service quality and plan resources in banking settings, where it's not always possible to get exact estimates of parameters.

**Abdullah, Ismail, and Osman (2016)** conducted a case study at Malaysian airport check-in counters to evaluate the effectiveness of fuzzy queueing models compared to traditional probabilistic models in handling uncertainties such as unpredictable customer arrival patterns and service times. The study addresses a common challenge in service operations, where arrival and service rates often fluctuate and lack precise estimation, particularly during peak travel periods. While the conventional M/M/1 model relies on fixed input parameters and exponential distributions, the fuzzy queueing model incorporates triangular fuzzy numbers to

capture expert judgment and imprecise observations. By applying fuzzy mathematics and  $\alpha$ -cut analysis, the study estimates key performance measures including average waiting time, queue length, and server utilization, and compares them with those obtained from classical models. The findings reveal that the fuzzy queueing model offers greater accuracy and adaptability, especially in scenarios with incomplete or ambiguous data. The authors conclude that fuzzy modeling provides a more effective approach for analyzing human-centric, dynamic input conditions in complex public service environments such as airports, highlighting the superiority of linguistic decision-making tools over rigid stochastic assumptions.

**Popovic, Simic, and Milinkovic (2018)**, in their paper titled “*ANFIS Model for Queuing in Warehouses*” published in MDPI, proposed a hybrid intelligent approach for optimizing job scheduling in warehouse queueing systems by integrating neural networks with fuzzy logic. The study employs an Adaptive Neuro-Fuzzy Inference System (ANFIS) to model the non-linear and uncertain dynamics inherent in warehouse operations, such as fluctuating task arrivals, processing times, and job priorities. Inputs including task priority, processing time, and queue length are fed into a Sugeno-type fuzzy inference system, enhanced through a neural learning algorithm that automatically tunes membership functions and rule bases based on training data. Simulation results and empirical validation demonstrate that the ANFIS model outperforms conventional scheduling techniques by reducing queue delays and optimizing resource utilization. By leveraging both machine learning and fuzzy reasoning, the model adapts to dynamic operational conditions while effectively handling imprecise input data. This research contributes significantly to the development of intelligent decision-support tools for queue management in complex and rapidly changing logistics environments, aligning with the advancements of Industry 4.0.

**Shanmugasundari and Aarthi's (2020)** work, “A Different Approach to Solve Fuzzy Queuing Theory,” shows a simple but useful way to look at queueing systems with fuzzy uncertainty. They focus on the standard M/M/1 queue model. The authors suggest a fuzzy extension of the M/M/1 model where both arrival rates ( $\lambda$ ) and service rates ( $\mu$ ) are shown as triangular fuzzy numbers. This is because systems in the real world often use rough or linguistic guesses instead of precise numbers. The study figures out key success indicators like the average length of a line and the average amount of time people have to wait by using fuzzy math. The method uses the extension principle and  $\alpha$ -cut techniques to make the

uncertainty go from input parameters to output measures in a planned way. Their findings show that fuzzy models are better than standard probabilistic methods at capturing the range and variability of how systems behave when there is uncertainty. This method is unique because it is easy to compute and understand, which makes it very useful for people who work in areas like traffic systems, transportation, and service operations where data isn't always accurate. This study adds to a growing amount of work that shows fuzzy logic can be useful in operational analysis and making decisions when there is uncertainty.

**Moghaddam, Fazlollahtabar, and Ghafourian (2021)** look into how fuzzy logic can be used to improve customer service evaluation in bank queueing systems in their work called "Evaluating the Bank Queuing Systems by Fuzzy Logic." This study presents a priority-based fuzzy decision-making framework that uses variables like customer credit score, account balance, and waiting time to dynamically prioritize service. This is different from traditional queueing models that treat all customers the same. The writers create a fuzzy inference system (FIS) with Mamdani-type fuzzy rules. They use membership functions and if-then fuzzy rules to figure out the service priority index for each customer based on their language inputs, such as "low balance," "long wait," and "good credit." This model lets banks choose how to handle customers based on less specific but still useful practical factors, rather than just the order of people in line. Results from simulations show that using fuzzy logic makes customers happier and makes better use of service resources, especially when there is a lot of traffic. The study shows how flexible fuzzy systems can be in customer-focused service settings. It also brings together fuzzy theory and real-world queue management in a useful way.

#### 4. Objectives

The primary purpose of this literature review is to critically investigate and synthesize the existing research on queueing models that were established on the basis of fuzzy numbers as a tool to manage uncertainty in service systems. In order to have an understanding of the development of fuzzy queueing theory, it is necessary to investigate essential models that combine fuzzy set theory with traditional queueing frameworks (for example, M/M/1 and M/M/C). Determine the many sorts of fuzzy numbers, such as triangular and trapezoidal, as well as membership functions that are typically utilized to express imprecise factors, such as arrival and service rates. In this context, it is important to emphasize the quantitative and computational tools that are utilized for analysis. These techniques include fuzzy arithmetic,  $\alpha$ -cut techniques, simulation, and hybrid approaches such as ANFIS and fuzzy inference

systems. Compare fuzzy models to classic probabilistic models, particularly with regard to their capacity to deal with data that is ambiguous, linguistic, or incomplete in a variety of fields, including finance, logistics, warehousing, and construction, among others. By analyzing case studies and performance outcomes, you can determine whether or not these models are applicable to real-world circumstances and whether or not they successfully perform. In particular, with regard to scalability, interpretability, and the incorporation of fuzzy models with emerging technologies such as artificial intelligence and the internet of things, it is important to identify research gaps, limitations, and future prospects. In the event that these aims are accomplished, the study will lay a strong groundwork for the subsequent research and practical use of fuzzy queueing systems in contexts that are fraught with uncertainty.

## 5. Methodology

**$\alpha$ -cut:** An  $\alpha$ -cut (also written as alpha-cut) of a fuzzy set  $\tilde{A}$  is a crisp set that includes all elements of the universe whose membership degree in  $\tilde{A}$  is greater than or equal to a specific threshold  $\alpha$ , where  $\alpha \in [0, 1]$ . Formally, for a fuzzy set  $\tilde{A}$  with membership function  $\mu_{\tilde{A}}(x)$ , the  $\alpha$ -cut is defined as:

$$A_{\alpha} = \{ x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha \}$$

where,  $X$  is the universe of discourse;  $A_{\alpha}$  is a crisp (non-fuzzy) subset of  $X$ .

**Triangular Fuzzy Number (TFN):** Defined by three real numbers: (a, b, c)

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a \\ \frac{x - a}{b - a}, & a < x \leq b \\ \frac{c - x}{c - b}, & b < x < c \\ 0, & x \geq c \end{cases}$$

- a: lower limit
- b: peak (mode, where  $\mu = 1$ )
- c: upper limit

**Trapezoidal Fuzzy Number (TrFN):** Defined by four real numbers: (a, b, c, d)

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a \\ \frac{x - a}{b - a}, & a < x \leq b \\ 1, & b < x \leq c \\ \frac{d - x}{d - c}, & c < x \leq d \\ 0, & x > d \end{cases}$$

- Used when there's a plateau (flat peak) in the 'most likely' region.

**DSW algorithm:** The DSW algorithm is primarily used to convert a fuzzy number into an interval-valued number at a specific  $\alpha$ -cut level. This conversion allows the use of interval

arithmetic to perform operations on fuzzy numbers, such as addition, subtraction, multiplication, and division.

### Queuing Theory Formulas (M/M/1 Model):

$\lambda$  = Arrival rate (customers per time unit)

$\mu$  = Service rate (customers per time unit)

$\rho = \frac{\lambda}{\mu}$  = Traffic intensity (must be  $< 1$  for stable system)

$L_s$  – Average number of customers in the system (including those being served):

$$L_s = \frac{\lambda}{(\mu - \lambda)}$$

$L_q$  – Average number of customers in the queue (waiting only):

$$L_q = \frac{\lambda^2}{[\mu(\mu - \lambda)]} = \frac{\rho^2}{(1 - \rho)}$$

$W_s$  – Average time a customer spends in the system (waiting + service):

$$W_s = \frac{1}{(\mu - \lambda)}$$

$W_q$  – Average time a customer spends waiting in the queue (before service):

$$W_q = \frac{\lambda}{[\mu(\mu - \lambda)]} = \frac{\rho}{(\mu - \lambda)}$$

### 6. Future Scope:

Fuzzy queueing methods offer various potential research directions despite substantial advances. In hybrid models, fuzzy logic is combined with additional uncertainty modeling techniques like stochastic processes, Bayesian networks, and machine learning (e.g., ANFIS or deep learning-based fuzzy controllers). Dynamic Fuzzy Parameters defines models with dynamic arrival and service rates stated as time-dependent fuzzy functions, rather than static fuzzy integers. Application Expansion explores new sectors like smart mobility, cloud computing, IoT-based services, cybersecurity, and pandemic response modeling with fuzzy queues. Real-Time Simulation and Control employs fuzzy logic for real-time decision-making in queue control systems (e.g., hospitals, call centers) using simulation platforms or real-time data. MCDM: Applying fuzzy queueing theory to balance cost, time, and service levels in uncertain multi-objective optimization settings. Tool Development creates open-source fuzzy queueing simulation tools or libraries in Python or MATLAB for better uptake in academia and industry.

### 7. Conclusion:

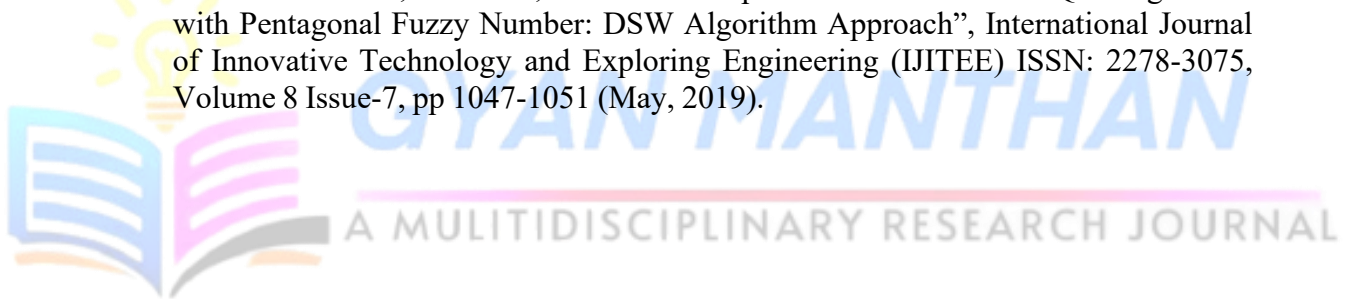
An important step forward in the modeling of service systems with intrinsic uncertainty, fuzziness, or imprecision has been fuzzy queueing models. By adding linguistic variables and

fuzzy numbers, fuzzy models provide a more realistic framework for queueing theory than classical queueing theory, which presupposes accurate knowledge of input parameters. The main methodologies utilized in the development and analysis of fuzzy queueing systems, as well as their various practical applications, have been covered in this survey. The widespread use of fuzzy methods in fields as diverse as medicine, finance, logistics, and communications attests to their adaptability and usefulness. By incorporating expert judgment and human reasoning into the analysis, these models not only improve decision-making under uncertainty, but they also provide system designers a leg up. Additional study is encouraged in the areas of adaptive control strategies, simulation-based optimization, hybrid fuzzy-stochastic models, and computational tools and fuzzy modeling methodologies. There is little doubt that future research in both academia and industry should focus on finding ways to incorporate fuzzy logic into queueing theory.

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***GYAN MANTHAN***

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